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AN APPLICATION OF SOMBOR INDEX OF HESITANCY FUZZY GRAPHS IN MOBILE NETWORKS

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Abstract: Pathinathan T., J. Jon Arockiaraj, and J. Jesintha Rosline [5] introduced the concept of Hesitancy Fuzzy Graphs (HFGs). In this work, we discuss an application of the Sombor index of a HFG in mobile networks. This work includes the comparison of the performance levels of mobile companies — Jio, BSNL, Airtel, and Vodafone Idea — in a selected area. Then, we investigate how these mobile companies can improve their performance.

Keywords and Phrases: Hesitancy Fuzzy Graphs, Degree of a vertex in a HFG, Sombor Index.

2020 Mathematics Subject Classification: 05C72, 05C76.

1. Introduction

Fuzzy sets were introduced by Lotfi A. Zadeh [10]. As an extension of this concept, Intuitionistic fuzzy sets were introduced by Krassimir Atanassov [2]. Later, Hesitant Fuzzy Sets were introduced by Vicenç Torra [8]. Azriel Rosenfeld [7] introduced Fuzzy Graph which is an extension of classical graph theory that incorporate uncertainty in relationships between nodes. Building on the foundations of Fuzzy Graphs and Hesitant Fuzzy Sets, Hesitancy Fuzzy Graphs (HFGs) were developed by T. Pathinathan and J. Jon Arockiaraj [5], to further refine the representation of uncertainty in graph structures. N. Vinothkumar and G. Geetharamani [9] have made significant contributions to HFG theory.

Topological indices are numerical invariants of a graph that characterize its structural properties. Sombor Index in graphs was introduced by I. Gutman [3]. Recent studies have demonstrated the applicability of Sombor indices in uncertain and fuzzy environments. For instance, S. Anwar, M. Azeem, M. K. Jamil, B. Almohsen, and Y. Shang [1] applied single-valued neutrosophic fuzzy Sombor numbers to analyze trade flows between countries via sea routes. Similarly, M. K. Jamil, S. Anwar, M. Azeem, and I. Gutman [4] utilized intuitionistic fuzzy Sombor indices to improve the performance of vaccination centers. In this paper, we discuss a practical application of the Sombor index of HFGs in the context of mobile networks.

1.1. Sombor Index of a HFG

Let $G(V, E) = (V(\mu_1, \gamma_1, \beta_1), E(\mu_2, \gamma_2, \beta_2)$ be a HFG. Then,

1. Membership sombor index $SO_\mu(G)$ is defined as
$SO_\mu(G) = \sum_{uv\in E} \sqrt{(\mu_1(u)d_\mu(u))^2 + (\mu_1(v)d_\mu(v)^2)}$.

2. Non-membership sombor index $SO_\gamma(G)$ is defined as
$SO_\gamma(G) = \sum_{uv\in E} \sqrt{(\gamma_1(u)d_\gamma(u))^2 + (\gamma_1(v)d_\gamma(v)^2)}$.

3. Hesitancy sombor index $SO_\beta(G)$ is defined as
$SO_\beta(G) = \sum_{uv\in E} \sqrt{(\beta_1(u)d_\beta(u))^2 + (\beta_1(v)d_\beta(v)^2)}$.

The Sombor index of the Hesitancy Fuzzy Graph G is defined as
$SO(G) = (SO_\mu(G), SO_\gamma(G), SO_\beta(G))$.

2. Main results

An application of Sombor index, which deals with the case of mobile towers, is discussed here.

2.1. Problem. To compare the performance of the mobile companies such as Jio, BSNL, Airtel and Vodafone-Idea (VI), using Sombor index of HFGs and to provide a way to improve the performance of the weaker one.

2.2. Methodology

We consider all 15 mobile towers in Poovam area in Changanacherry Municipality, Kottayam District, Kerala, India and we denote these towers by $t_1, t_2, ...t_{15}$.

Mobile companies such as Jio, BSNL, Airtel and Vodafone-Idea(VI) are offering services through these 15 towers. The data such as highest value of frequency used (f_h), lowest value of frequency used (f_l), highest generation network transmitted (hG) and the percentage of electromagnetic radiation emitted(r_e) are collected from the website.

We draw 4 HFGs representing these mobile companies, say J, B, A, V corresponding to Jio, BSNL, Airtel and VI respectively and find their Sombor indices. These are done using Python programs. The vertex of the HFG represents the mobile towers which provide the service of the mobile company under consideration. Two vertices are adjacent if and only if they are in a 1 km displacement(we can choose any distance for adjacency. Here we consider this distance as 1km). For constructing the HFGs, the membership, non-membership and hesitancy values are calculated as follows:

Membership Value (μ_1)

The membership value is calculated as $\mu_1 = f_h + f_l + \nu(hG)$, where f_h and f_l represent the highest and lowest frequency bands used respectively, and $\nu(hG)$ is the value corresponding to the highest generation network transmitted.

Highest Generation Network Transmitted(hG)	$\nu(hG)$
2G	0.1
3G	0.2
4G	0.3
5G	0.4

Table 1: Values Assigned for Highest Generation Network Transmitted

Hesitancy Value (β_1)

The hesitancy value is taken as the value corresponding to the percentage of electromagnetic radiation emitted by the tower.

Range of Radiation Emission (%)	**Hesitancy Value** β_1
0–20	0.10
20–40	0.15
40–60	0.20
60–80	0.25
80–100	0.30

Table 2: Values Assigned for Highest Generation Network Transmitted

Non-Membership Value (γ_1)

The non-membership value is defined as
$\gamma_1 = 1 - (\mu_1 + \beta_1)$.

Membership, Non-membership and Hesitancy values of Edges

Next, we define the membership, non-membership, and hesitancy values of the edges. For an edge $e = uv$, the membership, non-membership and hesitancy values are defined as: $\mu_2 = \min\{\mu_1(u), \mu_1(v)\}$, $\gamma_2 = \max\{\gamma_1(u), \gamma_1(v)\}$, $\beta_2 = \min\{\beta_1(u), \beta_1(v)\}$.

If the membership, non-membership, and hesitancy values are based on the performance of the network, then the Sombor index of the HFG represents the overall performance of the corresponding mobile company. Hanumantha Reddy D T, M V Chakradhara Raoand S. M. Hosamani [6] established that an increase in the Sombor index of a fuzzy graph corresponds to an improvement in the efficiency of the underlying system. We adopt this result as the theoretical foundation for this application.

The membership, non-membership, and hesitancy values of the vertices of the four mobile companies are calculated accordingly and listed in tables 3 to 6.

JIO	Highest Generation Network Transmitted	G	f_h	f_i	r_e	Membership Value ($f_h + f_i + G$)	Non-Membership Value	Hesitancy Value
t_1	4G	0.3	0.23	0.08	10.55	0.61	0.29	0.10
t_2	4G	0.3	0.23	0.08	7.73	0.61	0.29	0.10
t_3	4G	0.3	0.23	0.08	11.16	0.61	0.29	0.10
t_4	4G	0.3	0.23	0.08	20.5	0.61	0.24	0.15
t_7	4G	0.3	0.23	0.08	55.07	0.61	0.19	0.20
t_8	4G	0.3	0.23	0.08	33.3	0.61	0.24	0.15
t_9	4G	0.3	0.23	0.08	36.3	0.61	0.24	0.15
t_{10}	4G	0.3	0.23	0.08	52.5	0.61	0.19	0.20
t_{12}	4G	0.3	0.23	0.08	9.35	0.61	0.29	0.10
t_{15}	4G	0.3	0.23	0.00	4.9	0.53	0.37	0.10

Table 3: JIO: Membership, Non-membership and Hesitancy Values

BSNL	Highest Generation Network Transmitted	G	f_h	f_i	r_e	Membership	Non-Membership	Hesitancy
t_2	4G	0.3	0.21	0.07	7.73	0.58	0.32	0.10
t_3	4G	0.3	0.21	0.07	11.16	0.68	0.22	0.10
t_6	4G	0.3	0.21	0.07	57.54	0.58	0.22	0.20
t_7	3G	0.2	0.21	0.09	55.07	0.41	0.39	0.20
t_8	3G	0.2	0.21	0.09	33.39	0.53	0.35	0.15
t_{11}	4G	0.3	0.21	0.07	72.82	0.58	0.17	0.25
t_{14}	4G	0.3	0.21	0.07	17.7	0.58	0.32	0.10

Table 4: BSNL: Membership, Non-membership and Hesitancy Values

AIRTEL	Highest Generation Network Transmitted	G	f_h	f_i	r_e	Membership	Non-Membership	Hesitancy
t_2	4G	0.3	0.23	0.09	7.73	0.62	0.28	0.10
t_3	4G	0.3	0.23	0.09	11.16	0.62	0.28	0.10
t_5	5G	0.4	0.33	0.09	11.25	0.82	0.08	0.10
t_6	4G	0.3	0.23	0.09	57.54	0.62	0.18	0.20
t_7	4G	0.3	0.23	0.09	55.07	0.62	0.18	0.20
t_8	4G	0.3	0.23	0.09	33.39	0.62	0.23	0.15
t_9	4G	0.3	0.23	0.09	36.38	0.62	0.23	0.15
t_{13}	4G	0.3	0.23	0.09	5.05	0.62	0.28	0.10

Table 5: AIRTEL: Membership, Non-membership and Hesitancy values

VI	Highest Generation Network Transmitted	G	f_h	f_i	r_e	Membership	Non-Membership	Hesitancy
t_2	4G	0.3	0.25	0.09	7.73	0.64	0.26	0.10
t_3	4G	0.3	0.18	0.09	11.16	0.57	0.33	0.10
t_5	4G	0.3	0.18	0.09	11.25	0.57	0.33	0.10
t_7	4G	0.3	0.25	0.09	55.07	0.64	0.16	0.20
t_8	4G	0.3	0.25	0.09	33.39	0.64	0.21	0.15
t_9	4G	0.3	0.25	0.09	36.38	0.64	0.21	0.15
t_{13}	4G	0.3	0.18	0.09	5.05	0.57	0.33	0.10

Table 6: Vodafone Idea: Membership, Non-membership and Hesitancy values

Note: The corresponding HFGs are drawn in figures 1 to 4. In these HFGs, the vertex t_2 is isolated because its displacement from the other vertices is more than

1 km.

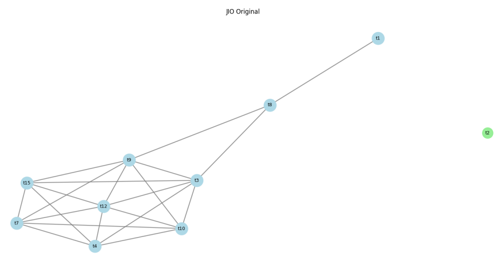

Figure 1: Jio Original

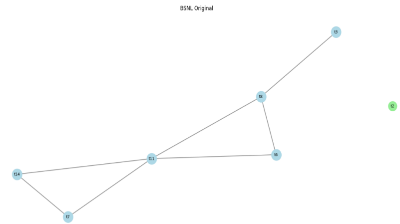

Figure 2: BSNL Original

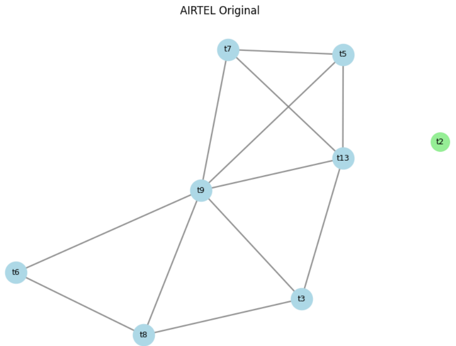

Figure 3: Airtel Original

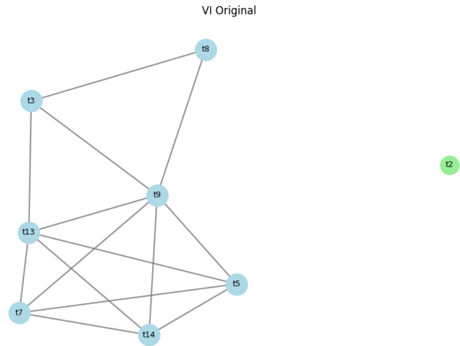

Figure 4: Vodafone idea Original

Now, the Sombor index of the HFGs J, B, A and V are calculated. Python is utilized to calculate the Sombor indices and to draw the hesitancy fuzzy graphs in each case. The Sombor indices obtained as:

HFGs	**Membership Value**	**Non-Membership Value**	**Hesitancy Value**
J	55.43	12.44	2.69
B	7.81	2.51	0.90
A	26.64	3.98	1.15
V	18.62	4.30	0.80

Table 7: Sombor Index of the Four HFGs

By analyzing table 7, we observed that Jio shows the highest performance level compared to the other mobile companies because the HFG J has the highest membership Sombor index. If the membership Sombor indices are very close, we compare the hesitancy Sombor index and the one with less hesitancy Sombor index is chosen as the best choice.

2.3. Method of Improving the weaker Mobile Companies

We found that JIO has the highest performance level. Next, we investigate how the BSNL company can attain performance levels near to that of Jio. We can achieve this by adding each combination of vertices, not included in each company's network, and then finding the Sombor Index. We developed a Python program to consider the possible combination of towers where there are no BSNL antennas yet and find the sombor index of the resulting graph. The Python program accepts two input values—target membership value and range value. The target membership value is the membership Sombor index of the highly performed mobile company, and the range value is a small positive number (ϵ), which we are adding and subtracting from the target membership value (T) to get the range. This program outputs all such combinations where the membership Sombor index falls between the target membership value ± range value, i.e., in the range $(T-\epsilon, T+\epsilon)$. Here we choose T = 55.5 and ϵ= 0.5. Therefore, the membership Sombor Index range is (55, 56). So, we get the output as all such combinations where the membership Sombor index falls in the range of 55 to 56. The following table gives the combination of towers, which, when are added to the HFG B, gives the membership Sombor index within a range (55, 56).

Tower	**Membership Value**	**Non-Membership Value**	**Hesitancy Value**
t_1	0.58	0.32	0.10
t_4	0.58	0.27	0.15
t_5	0.58	0.32	0.10
t_9	0.58	0.27	0.15
t_{10}	0.58	0.22	0.20
t_{12}	0.58	0.32	0.10
t_{13}	0.58	0.32	0.10
t_{15}	0.58	0.32	0.10

Table 8: Newly Assigned BSNL Values for Improvement

Sl. No.	**Towers in which Antennas are added**	**Membership Sombor index**	**Non-Membership Sombor index**	**Hesitancy Sombor index**
1	t_9, t_{10}, t_{12}	55.34	16.87	4.08
2	t_9, t_{10}, t_{13}	55.34	16.87	4.08
3	t_1, t_4, t_9, t_{12}	55.29	17.44	4.04
4	t_1, t_4, t_9, t_{13}	55.29	17.44	4.04

Table 9: Combinations of Towers for BSNL near to Jio's Performance

We choose the value near to 55.43 with minimum hesitancy value and the minimum number of towers. Here the first and second combination is found as the best option. So if BSNL install antennas in towers t_9, t_{10} and t_{12} or t_9, t_{10} and t_{13}, they can attain the performance level near to Jio.**So one of these two are the best choices to improve the performance of BSNL similar to that of Jio.** The HFG B_1 after installing antennas in towers t_9, t_{10} and t_{12} to B and the HFG B_2 after installing antennas in towers t_9, t_{10} and t_{13} to B are given in figures 5 and 6:

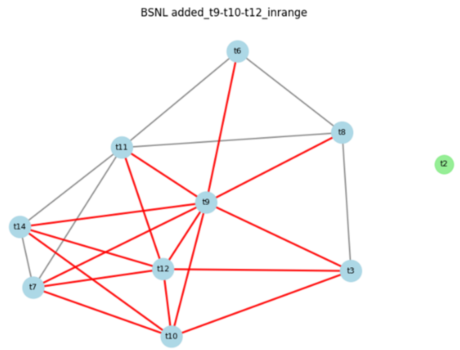

Figure 5: B_1

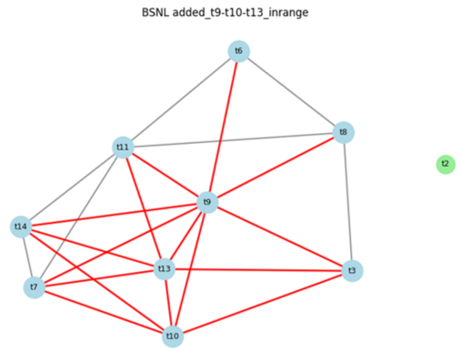

Figure 6: B_2

Now, apply the same procedure for Airtel and Vodafone Idea. **In the case of Airtel four combinations are obtained within the chosen range 55 to 56.** The sombor indices of the corresponding HFGs in these two cases are given in tables 10 and 11. In the case of Airtel, after comparing the Membership values, we find that three graphs have the same value. Hence, we compare the Hesitancy values and choose the graph with the lowest hesitancy. Here, 2^{nd} and 4^{th} graphs show the maximum performance. The HFG A_1 after installing antennas in towers t_4 and $t_{12}$to A and the HFG A_2 after installing antennas in towers t_4 and t_{15} to A are given in figures 7 and 8.

In the case of Vodafone Idea, the third and fourth combinations are found as the best option. The HFGs are given in figures 9 and 10.

Sl. No.	Vertices Added	Membership Value	Non-Membership Value	Hesitancy Value
1	t_4, t_{10}	55.73	8.57	2.72
2	t_4, t_{12}	55.73	9.31	2.24
3	t_4, t_{14}	55.70	8.93	2.23
4	t_4, t_{15}	55.73	9.31	2.24

Table 10: Combinations of Towers for Airtel near to Jio's Performance

Sl. No.	Vertices Added	Membership Value	Non-Membership Value	Hesitancy Value
1	t_6, t_{10}, t_{15}	55.42	11.05	2.61
2	t_1, t_4, t_6, t_{10}	55.36	10.73	2.83
3	t_1, t_4, t_6, t_{12}	55.36	11.37	2.35
4	t_1, t_4, t_6, t_{15}	55.36	11.37	2.35

Table 11: Combinations of Towers for VI near to Jio's Performance

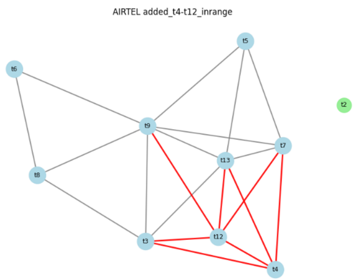

Figure 7: A_1

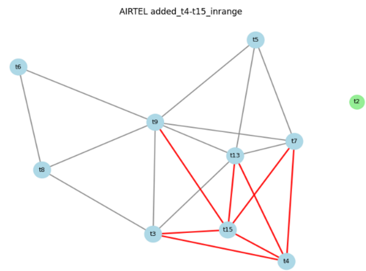

Figure 8: A_2

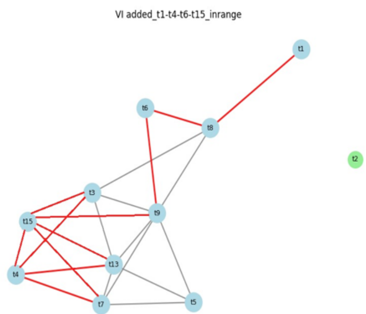

Figure 9: V_1

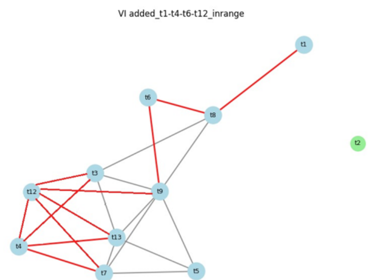

Figure 10: V_2

3. Conclusion

In this work, we have compared the performance of mobile companies by calculating their *Sombor Indices*. After comparing these indices, we proposed a method to improve the performance of the lower-performing companies. Furthermore, we attempted to raise the performance level of other providers to be on par with the highest-performing provider (in this case, JIO).For all these processes, we developed Python programs, which significantly reduce the manpower required to perform such calculations.

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